

Unit - III

Combinational logic Circuits

Half Adder:

→ Half adder

→ full adder

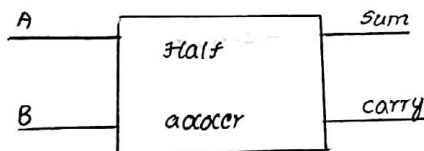
→ Ripple adder / Parallel adder

→ carry look ahead adder

Half adder

used to add 2 bits

Block diagram:



Truth table

Inputs		Outputs	
A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

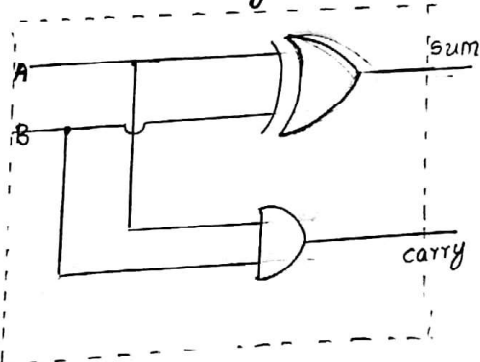
logic expressions:

$$\text{Sum} = \bar{A}B + A\bar{B}$$

$$= A \oplus B$$

$$\text{Carry} = A \cdot B$$

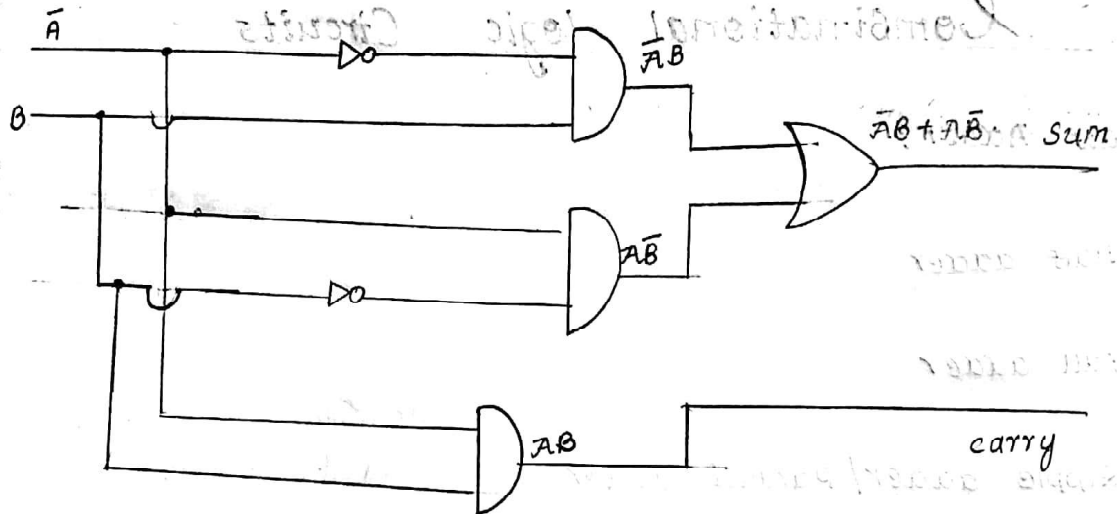
logic circuit:



Half Adder using Basic gates

$$\text{Sum} = \bar{A}B + A\bar{B}$$

$$\text{Carry} = AB$$



Half adder using N gate Nand gate

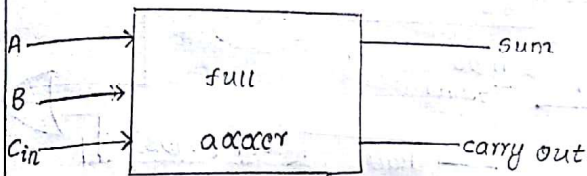
Truth Table

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Full adder: used to add 2 input bits along with carry

input

Truth Table



inputs			Outputs	
A	B	Cin	sum	carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$\text{sum} = \sum m(1, 2, 4, 7)$$

$$\text{carry} = \sum m(3, 5, 6, 7)$$

Sum:-

$$= \bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in}$$

$$= [C_{in}(\bar{A}\bar{B} + AB)] + [\bar{C}_{in}(\bar{A}B + A\bar{B})]$$

$$= [C_{in}(\overline{A \oplus B})] + [C_{in}(A \oplus B)]$$

$$= C_{in} \oplus (A \oplus B)$$

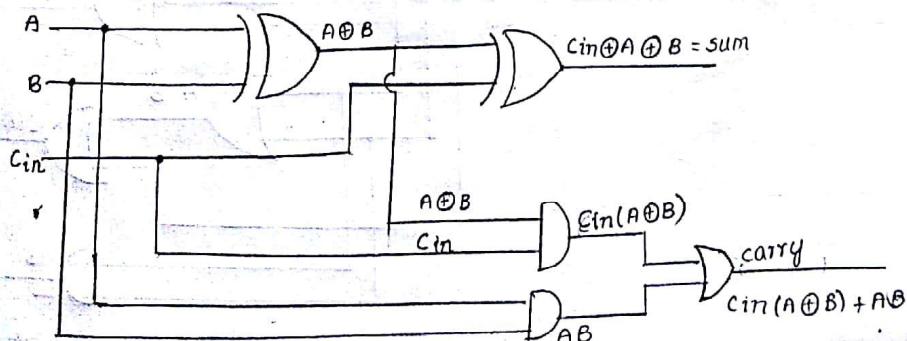
$$= C_{in} \oplus A \oplus B$$

$$\text{Carry:- } \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + AB\bar{C}_{in} + ABC_{in}$$

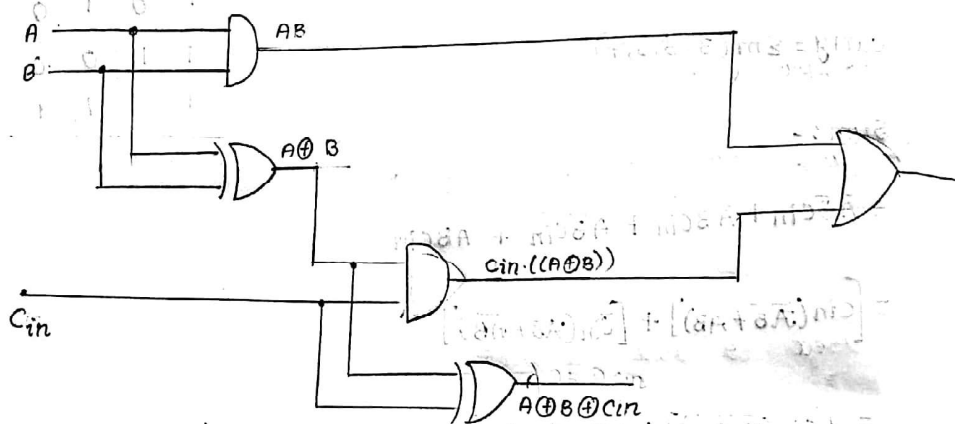
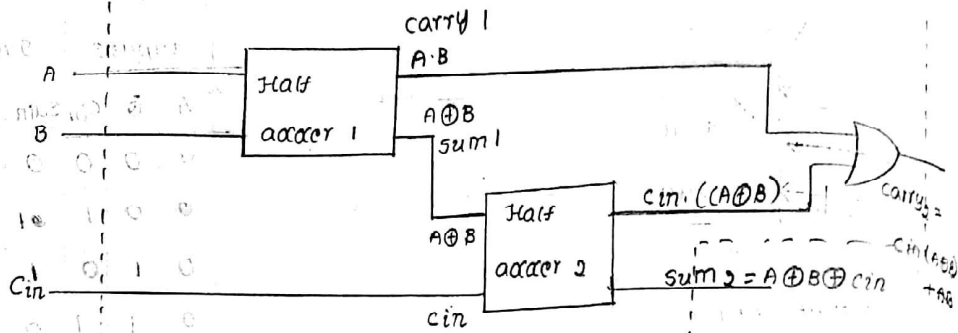
$$= C_{in}(\bar{A}B + A\bar{B} + AB) + AB\bar{C}_{in}$$

$$= C_{in}(\bar{A}B + A\bar{B} + AB) + AB(C_{in} + \bar{C}_{in})$$

$$= C_{in}(A \oplus B) + AB$$



Full adder using two half adders and one OR gate



$$Sum = \sum m(1, 2, 4, 7)$$

$$Carry = \sum m(3, 5, 6, 7)$$

$\bar{A} \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	1	3	2
A	4	5	7	6

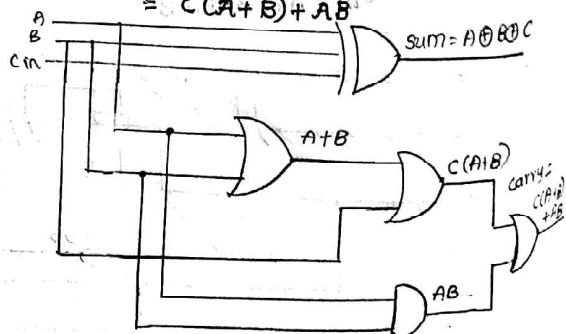
$\bar{A} \backslash BC$	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	0	0	1	0
A	0	1	1	1

$$= \bar{A}\bar{B}c_{in} + \bar{A}B\bar{c}_{in} + A\bar{B}\bar{c}_{in} + ABC_{in} = (\bar{A} + A)BC + A(\bar{B}C + BC) + A(B\bar{C} + \bar{B}C)$$

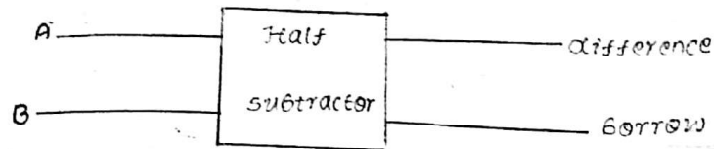
$$= c_{in}(\bar{A}\bar{B} + AB) + \bar{c}_{in}(\bar{A}B + A\bar{B}) = BC + AC + AB$$

$$= c_{in}(\bar{A} \quad c_{in} \oplus A \oplus B$$

$$= C(A+B) + AB$$



Half subtractor : used to subtract 2 bits



Truth Table

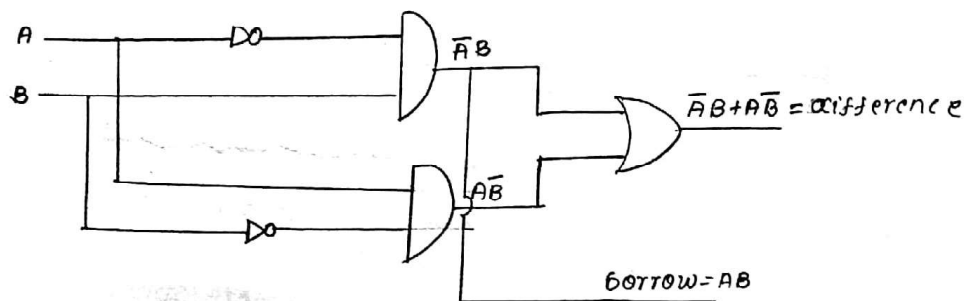
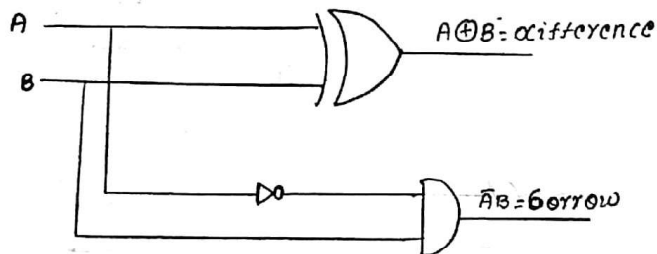
Inputs		Output	
A	B	diff	borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$\text{Difference} = \bar{A}B + A\bar{B}$$

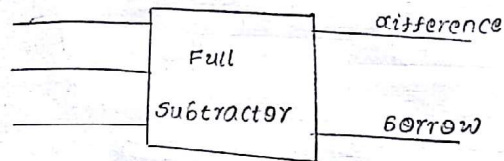
$$\text{Borrow} = \bar{A}B$$

$$= A \oplus B$$

Logic circuit:



Full subtractor:



Truth Table

Inputs			Outputs	
A	B	Bin	diff	borrow
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	0	1

$$\text{diff} = \bar{A}\bar{B}B_{in} + \bar{A}\bar{B}\bar{B}_{in} + A\bar{B}\bar{B}_{in} + AB\bar{B}_{in}$$

$$+ AB B_{in}$$

$$= B_{in}(\bar{A}\bar{B} + AB) + \bar{B}_{in}(\bar{A}\bar{B} + AB)$$

$$= B_{in}(\overline{A \oplus B}) + \bar{B}_{in}(A \oplus B)$$

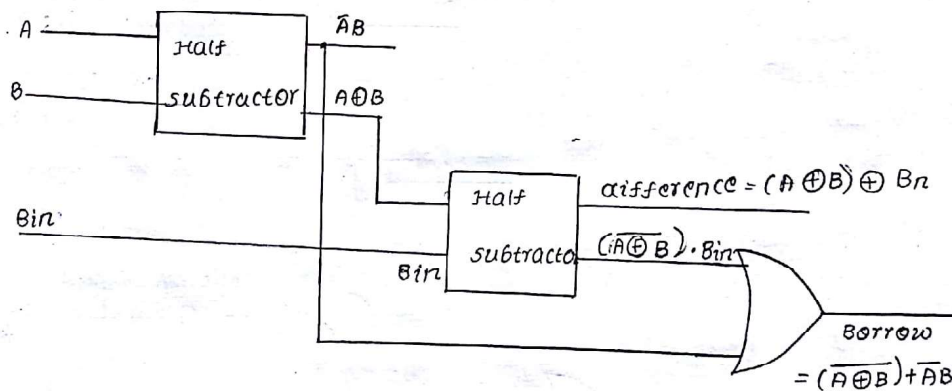
$$A \oplus B + \bar{A}\bar{B}$$

$$= B_{in} \oplus A \oplus B$$

$$\text{carry} = \text{borrow} = \bar{A}\bar{B}B_{in} + \bar{A}B\bar{B}_{in} + \bar{A}B B_{in} + A\bar{B}\bar{B}_{in}$$

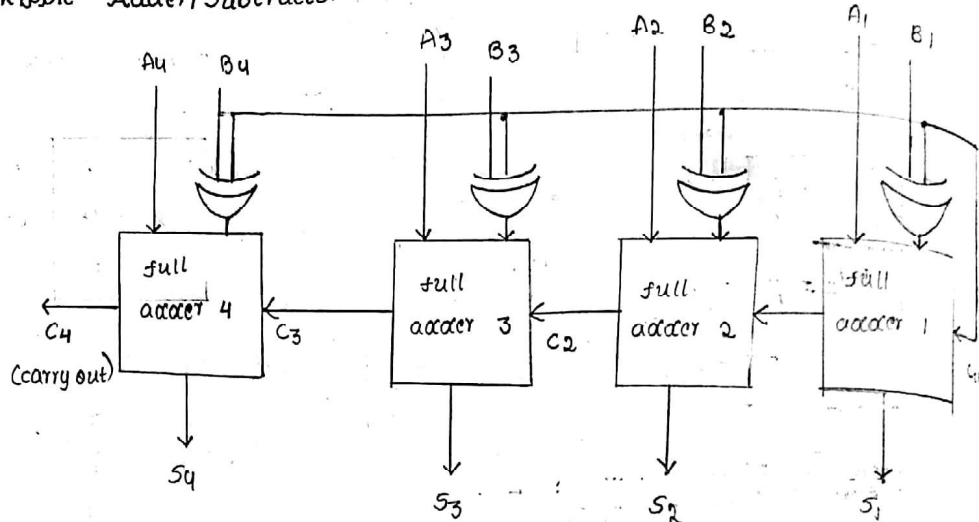
$$= B_{in}(\bar{A}\bar{B} + \bar{A}B + AB) + \bar{A}\bar{B}\bar{B}_{in} \quad B_{in}(\bar{A}\bar{B} + \bar{A}B) + \bar{A}\bar{B}$$

$$= B_{in}(\overline{A \oplus B}) + \bar{A}\bar{B}$$



Block diagram of full subtractor using 2 half subtractors.

Ripple Adder/Subtractor



$$B \oplus 0 = B$$

$$B \oplus 1 = \overline{B}$$

$$B = 0 \text{ or } 1$$

$C_{in} = 0 \rightarrow$ addition

$= 1 \rightarrow$ subtraction in 2's complement form

X5-3 Addition: Adder:

1. To perform XS-3 addition we have to add 2 XS-3 code groups.
2. If carry = 1 add 0011 (3) to the sum of those group
3. If carry = 0 add 1101 (13) to the sum of those groups

Eg 1:- $7+5=12$

Eg:- $4 + 3 = 7$

$$\begin{array}{r}
 1010 \\
 \underline{1000} \\
 00010010 \\
 0011+0011 \\
 \hline
 0100 \quad 0101 \\
 \begin{array}{cc}
 1 & 2 \\
 \underbrace{\hspace{1cm}} & \\
 \times 5-3
 \end{array}
 \end{array}$$

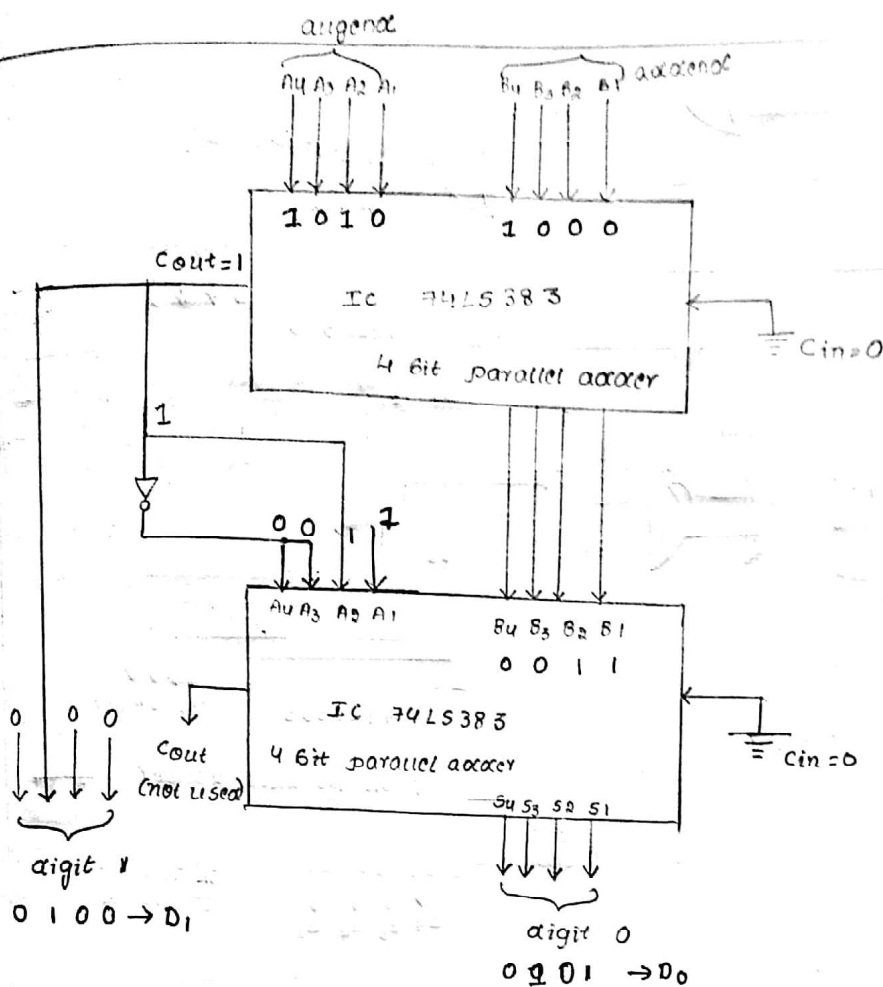
$$4 \xrightarrow{x5-3} 0111$$

3 XS-3 → 0110

1151
1151

$$\begin{array}{r} 1101 \\ \hline 71010 \end{array}$$

discards is $x5-3$



BCD Adder (4 bit);

Eg1:-

$$\begin{array}{r}
 5 \rightarrow 0101 \\
 6 \rightarrow 0110 \\
 \hline
 11 \rightarrow 1011 \\
 + 0110 \\
 \hline
 0001 \quad 0001 \\
 \text{digit 1} \quad \text{digit 0}
 \end{array}$$

$$\begin{array}{r}
 4 \rightarrow 0100 \\
 3 \rightarrow 0011 \\
 \hline
 7 \rightarrow 0111 \rightarrow < 9 \\
 0000 \\
 \hline
 0111 \rightarrow 7 \text{ in BCD}
 \end{array}$$

C ₃	C ₂	C ₁	C _{in}
A ₄	A ₃	A ₂	A ₁
B ₄	B ₃	B ₂	B ₁
S ₅	S ₄	S ₃	S ₂
S ₁	S ₂	S ₃	S ₄

↑
carry out

$X = S_5 + S_4 (S_3 + S_2)$

Sum	S ₅	S ₄	S ₃	S ₂	S ₁
10	0	1	0	1	0
11	0	1	0	1	1
12	0	1	1	0	0
13	0	1	1	0	1
14	0	1	1	1	0
15	0	1	1	1	1
16	1	0	0	0	0
17	1	0	0	0	1
18	1	0	0	1	0

